CONTROL OF THE WEB LATERAL POSITION AND SLOPE BY EMPLOYING SPATIALLY DEPENDENT TRANSFER FUNCTIONS

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ABSTRACT

In this work, we employ a non-dimensional state space representation of the spatially dependent lateral transfer functions to study web lateral position behavior in response to lateral displacement and rotation of the guide roller as inputs. In particular, we will show that controlling the web lateral position and slope requires independently controlling the rotation and translation of the guide roller. We will discuss how the spatially dependent lateral transfer functions and measurements from several edge sensors may be utilized to obtain an estimate of the web slope. We will also discuss control strategies for controlling both web lateral position and slope and attenuation of lateral disturbances. We will provide results from numerical simulations of representative guiding situations to support the developments and discussions. This study is particularly relevant for high-precision lateral regulation within the span that may be required for emerging R2R applications in flexible and hybrid electronics such as nanoimprinting, printing, and deposition processes.

1 INTRODUCTION

Focused studies on modeling the lateral behavior of moving webs goes back to at least the 1960's. The first comprehensive work was reported by Shelton in his PhD thesis in 1968 [1], in which the moving web between two rollers is treated as a static Euler-Bernoulli tensioned beam. Four boundary conditions (web lateral position and slope on each roller) were utilized to solve the ordinary differential equation describing the web lateral position as a function of spatial distance along the span. Additionally, a key observation that is prevalent in the transport of belts literature was employed to describe how the web approaches the roller at the entry of the region of wrap; that is, a belt approaching a roller aligns itself normal to the axis of rotation of the roller. This normal entry rule was employed to develop two normal entry conditions, one for velocity and one for acceleration, at the entry of the web wrap on the roller. Then, transfer functions were derived from the guide roller position (input variable) to the web lateral position on the

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guide roller (controlled output variable). Using this approach, a number of studies performed analysis of lateral behavior and/or designed lateral controllers [2–7]. The developed controllers for controlling the web lateral position on the guide roller are based on the feedback of a sensor located immediately downstream the guide roller. A key deficiency of existing approaches is that one can only obtain lateral web position behavior on the roller. In [8] a guiding apparatus that can control the web lateral position and slope is presented; the approach is to use the reading of at least four web lateral sensors distributed along the span to solve simultaneously the required equations for the four unknown parameters in the general solution of the lateral governing equation, and more downstream sensors as feedback for the independent Proportional and Integral (PI) controllers for web position and slope, respectively.

Recently, [9–11] developed a method to obtain spatially dependent lateral transfer functions which were utilized to obtain the response of web lateral position and slope at any point within the web span and on the rollers. This method is based on applying the 1-D Laplace Transform in the temporal variable to the governing and redefined boundary conditions. These spatially dependent transfer functions may be employed to predict lateral behavior at any point in the web span due to guide roller movement and propagation of upstream disturbances, as well as to control the lateral position within a span and on the roller.

In this work, we employ a non-dimensional state space representation of the spatially dependent transfer functions, using the method presented in [12], to study the properties of the lateral system of equations with guide roller lateral displacement and rotation as the control inputs. We will use this analysis to show that controlling web lateral position and slope requires to independent control of the guide roller translation and rotation in the plane of the web. We will discuss how the spatially dependent lateral transfer functions and measurements from three edge sensors can be utilized to obtain an estimate of web slope; the placement of these three sensors is also discussed. We will also discuss control strategies for controlling both web lateral position and slope and attenuation of upstream lateral disturbances. Results from numerical simulations of representative guiding situations will be presented to support the developments and discussions. This study is particularly relevant for high-precision lateral regulation within the span that may be required for emerging R2R applications in flexible and hybrid electronics such as nanoimprinting, printing, and deposition processes.

The rest of this paper is organized as follows. A review of the development of spatially dependent lateral transfer functions is provided in Section 2. The nondimensional state-space representation, the closed loop dynamics using a Proportional and Integral (PI) controller, and the discussion of the effect of several control actions are presented in Section 3. In Section 5, an observer design is proposed for estimating the states of the system, including the web slope. Numerical simulations are presented in Section **??**. A summary of this work is provided in Section 6.

2 SPATIALLY DEPENDENT LATERAL TRANSFER FUNCTIONS

Spatially dependent lateral transfer functions for web lateral dynamics for typical situations were derived in [11]. The web lateral position and slope at any point within the span L are related to the lateral position at the entry of the span y_0, the rotation of the roller at the entry of the span θ_0; the lateral and rotational movements of the roller at the exit of the span, z_L and θ_L, respectively. Figure 1 illustrates a typical web span with these parameters. The spatially dependent

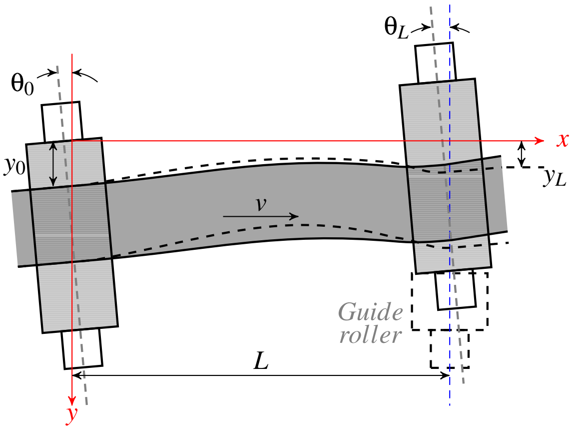

Figure 1 – Web span

transfer functions for web lateral position and slope are given by

$$\hat{y}(x,s) = \frac{P_4(x,s)}{D(s)}\hat{z}_L(s) + \frac{P_3(x,s)}{D(s)}\hat{\theta}_L(s) + \frac{P_1(x,s)}{D(s)}\hat{\theta}_0(s) + \frac{P_2(x,s)}{D(s)}\hat{y}_0(s) \qquad \{1\}$$

$$\frac{\partial \hat{y}}{\partial x}(x,s) = \frac{\frac{\partial P_4}{\partial x}(x,s)}{D(s)}\hat{z}_L(s) + \frac{\frac{\partial P_3}{\partial x}(x,s)}{D(s)}\hat{\theta}_L(s) + \frac{\frac{\partial P_1}{\partial x}(x,s)}{D(s)}\hat{\theta}_0(s) + \frac{\frac{\partial P_2}{\partial x}(x,s)}{D(s)}\hat{y}_0(s) \qquad \{2\}$$

where

$$\begin{aligned}
P_1(x,s) &= \frac{1}{g_2(L)}\left[\big((x-g_1(x))g_2(L)-(L-g_1(L))g_2(x)\big)s^2 + v\big(xg_1(L)-Lg_1(x)\big)\big)s + (x-g_1(x))v^2\right],\\
P_2(x,s) &= \frac{1}{g_2(L)}\left[(g_2(L)-g_2(x))s^2 + v(g_1(L)-g_1(x))s + v^2\right],\\
P_3(x,s) &= \frac{1}{g_2(L)}\left[(g_1(x)g_2(L)-g_1(L)g_2(x))s^2 + v^2 g_1(x)\right],\\
P_4(x,s) &= \frac{1}{g_2(L)}\left[g_2(x)s^2 + vg_1(x)s\right],\\
D(s) &= s^2 + v\frac{g_1(L)}{g_2(L)}s + \frac{v^2}{g_2(L)}, \quad \hat{y}_0(s) = \hat{z}_0(s) + \hat{y}_d(s),\\
g_1(x) &= \frac{\sinh(KL)[\cosh(Kx)-1] - \cosh(KL)[\sinh(Kx)-Kx]}{K[\cosh(KL)-1]},\\
g_2(x) &= \frac{[\cosh(KL)-1][\cosh(Kx)-1] - \sinh(KL)[\sinh(Kx)-Kx]}{K^2[\cosh(KL)-1]}.
\end{aligned}$$

To express the above equations in non-dimensional form, we define the following: $\varepsilon = x/L$; $\eta = y/L$, $\sigma_i = z_i/L$, $\bar{K} = TL^2/E_m I$,

$$\bar{g}_1(\varepsilon) = \frac{\sinh(\bar{K})[\cosh(\bar{K}\varepsilon)-1] - \cosh(\bar{K})[\sinh(\bar{K}\varepsilon)-\bar{K}\varepsilon]}{\bar{K}[\cosh(\bar{K})-1]},$$
$$\bar{g}_2(\varepsilon) = \frac{[\cosh(\bar{K})-1][\cosh(\bar{K}\varepsilon)-1] - \sinh(\bar{K})[\sinh(\bar{K}\varepsilon)-\bar{K}\varepsilon]}{\bar{K}^2[\cosh(\bar{K})-1]}.$$

The non-dimensional form of the governing equations for lateral position and slope are given by

$$\begin{aligned}\hat{\eta}(\varepsilon,s) = {} & \frac{a_4(\varepsilon)s^2 + b_4(\varepsilon)s}{s^2+bs+c}\hat{\sigma}_1(s) + \frac{a_3(\varepsilon)s^2+b_3(\varepsilon)s+c_3(\varepsilon)}{s^2+bs+c}\hat{\theta}_1(s) \\ & + \frac{a_1(\varepsilon)s^2+b_1(\varepsilon)s+c_1(\varepsilon)}{s^2+bs+c}\hat{\theta}_0(s) + \frac{a_2(\varepsilon)s^2+b_2(\varepsilon)s+c_2(\varepsilon)}{s^2+bs+c}\hat{\eta}_0(s) \qquad \{3\}\end{aligned}$$

$$\begin{aligned}\hat{\eta}_\varepsilon(\varepsilon,s) = {} & \frac{a_{4\varepsilon}(\varepsilon)s^2 + b_{4\varepsilon}(\varepsilon)s}{s^2+bs+c}\hat{\sigma}_1(s) + \frac{a_{3\varepsilon}(\varepsilon)s^2+b_{3\varepsilon}(\varepsilon)s+c_{3\varepsilon}(\varepsilon)}{s^2+bs+c}\hat{\theta}_1(s) \\ & + \frac{a_{1\varepsilon}(\varepsilon)s^2+b_{1\varepsilon}(\varepsilon)s+c_{1\varepsilon}(\varepsilon)}{s^2+bs+c}\hat{\theta}_0(s) + \frac{a_{2\varepsilon}(\varepsilon)s^2+b_{2\varepsilon}(\varepsilon)s+c_{2\varepsilon}(\varepsilon)}{s^2+bs+c}\hat{\eta}_0(s) \qquad \{4\}\end{aligned}$$

where the subscript ε represents the first derivative with respect to ε and

$$b = v\frac{\bar{g}_1(1)}{L\bar{g}_2(1)}, \quad c = \frac{v^2}{L^2\bar{g}_2(1)}, \quad a_1(\varepsilon) = \frac{(\varepsilon - \bar{g}_1(\varepsilon))\bar{g}_2(1) - (1-\bar{g}_1(1))\bar{g}_2(\varepsilon)}{\bar{g}_2(1)},$$
$$b_1(\varepsilon) = \frac{v(\varepsilon\bar{g}_1(1) - \bar{g}_1(\varepsilon))}{L\bar{g}_2(1)}, \quad c_1(\varepsilon) = \frac{v^2(\varepsilon\bar{g}_\varepsilon(1) - \bar{g}_1(\varepsilon))}{L^2\bar{g}_2(1)}, \quad a_2(\varepsilon) = \frac{\bar{g}_2(1) - \bar{g}_2(\varepsilon)}{\bar{g}_2(1)},$$
$$b_2(\varepsilon) = \frac{v(\bar{g}_1(1) - \bar{g}_1(\varepsilon))}{L\bar{g}_2(1)}, \quad c_2(\varepsilon) = \frac{v^2}{L^2\bar{g}_2(1)}, \quad a_3(\varepsilon) = \frac{\bar{g}_1(\varepsilon)\bar{g}_2(1) - \bar{g}_1(1)\bar{g}_2(\varepsilon)}{\bar{g}_2(1)},$$
$$b_3(\varepsilon) = 0, \quad c_3(\varepsilon) = \frac{v^2\bar{g}_1(\varepsilon)}{L^2\bar{g}_2(1)}, \quad a_4(\varepsilon) = \frac{\bar{g}_2(\varepsilon)}{\bar{g}_2(1)}, \quad b_4(\varepsilon) = \frac{v\bar{g}_1(\varepsilon)}{L\bar{g}_2(1)}.$$

This system consists of two outputs, lateral web position and slope (Eqns. {3} and {4}) that are related to four inputs which can be considered as either control actions or disturbances. Generally, $\hat{\eta}_0(s)$, $\hat{\theta}_0(s)$ are considered as the disturbances and the system is controlled by the lateral displacement ($\hat{\sigma}_1(s)$) and rotation ($\hat{\theta}_1(s)$) of the guide roller. In this work we will assume $\hat{\theta}_0(s) = 0$, which means that the entry roller is aligned. The goal of any controller is to reject the existing oscillations in the span, which means that the web lateral position and slope must be zero along the span [2]. However, in existing literature, because lateral transfer functions with lateral position output on the guide rollers were only available, the guide roller input was used to regulate only the position immediately downstream of the guide roller; the determination of the slope was considered irrelevant. Therefore, just one control action was required, which is the case for two commonly known intermediate guides, remotely pivoted guide (RPG) and offset pivot guide (OPG), where the roller rotation and displacement are kinematically constrained. Considering that the previous approach only accounts for the web lateral position on the rollers, the system can be viewed as a Single Input Single Output (SISO) system.

With spatially dependent transfer functions, we obtain a Multiple Input Multiple Output (MIMO) system, and if only one control action is applied, the system is reduced to two outputs (web lateral position and slope) and one input, which is an under-actuated system. To demonstrate this point, let us consider the following equation for an RPG guide, whose pivot point is at a distance X_1 from the exit roller,

$$\hat{\eta}(\varepsilon, s) = \frac{(a_3(\varepsilon) + \frac{X_1}{L} a_4(\varepsilon))s^2 + (\frac{X_1}{L} b_4(\varepsilon))s + c_3(\varepsilon)}{s^2 + bs + c} \hat{\theta}_1(s) + \frac{a_2(\varepsilon)s^2 + b_2(\varepsilon)s + c_2(\varepsilon)}{s^2 + bs + c} \hat{\eta}_0(s) \quad \{5\}$$

$$\hat{\eta}_\varepsilon(\varepsilon, s) = \frac{(a_{3\varepsilon}(\varepsilon) + \frac{X_1}{L} a_{4\varepsilon)}(\varepsilon))s^2 + (\frac{X_1}{L} b_{4\varepsilon}(\varepsilon))s + c_{3\varepsilon}(\varepsilon)}{s^2 + bs + c} \hat{\theta}_1(s) + \frac{a_{2\varepsilon}(\varepsilon)s^2 + b_{2\varepsilon}(\varepsilon)s + c_{2\varepsilon}(\varepsilon)}{s^2 + bs + c} \hat{\eta}_0(s) \quad \{6\}$$

Considering that regulation of the web lateral position at a spatial point ε_c in the span is desired, and the control action is the rotation of the guide roller that is generated by a Proportional and Integral (PI) controller, whose feedback is the web lateral position of the controlled point (ε_c), i.e., $\theta_1(s) = G_c(s)\hat{\eta}(\varepsilon_c, s)$, where $G_c(s)$ is the PI controller, the closed loop governing equations for web lateral position and slope are

$$\hat{\eta}(\varepsilon, s) = \frac{N_2(\varepsilon, s)(1 - (N_g(\varepsilon_c, s)G_c(s))) + N_g(\varepsilon, s)N_2(\varepsilon_c, s)G_c(s)}{(s^2 + bs + c)(1 - N_g(\varepsilon_c, s)G_c(s))} \hat{\eta}_0(s) \quad \{7\}$$

$$\hat{\eta}_\varepsilon(\varepsilon, s) = \frac{a_{2\varepsilon}(\varepsilon)s^2 + b_{2\varepsilon}(\varepsilon)s + c_{2\varepsilon}(\varepsilon)}{s^2 + bs + c} \hat{\eta}_0(s) + N_{g\varepsilon} G_c(s)\hat{\eta}(\varepsilon_c, s) \quad \{8\}$$

where $N_2(\varepsilon, s) = a_2(\varepsilon)s^2 + b_2(\varepsilon)s + c_2(\varepsilon)$; $G_c(s) = K_p + \frac{K_i}{s}$;
$N_g(\varepsilon, s) = a_3(\varepsilon) + \frac{X_1}{L} a_4(\varepsilon))s^2 + (\frac{X_1}{L} b_4(\varepsilon))s + c_3(\varepsilon)$; and $N_{g\varepsilon}$ is the derivative of N_g with respect to ε. From {7} and {8}, one can see that if the position at a given point $\hat{\eta}(\varepsilon_c, s)$ is regulated, then the slope cannot be controlled, because there is a residual error, given by the first term in the right hand side of Eq. {8}. Therefore, to control the web slope also, another independent control action is needed.

3 STATE SPACE REPRESENTATION

To facilitate the analysis of the system, we consider a state state representation of the spatially dependent lateral transfer functions. First, let us represent the system of transfer functions as:

$$\begin{pmatrix} \eta(\varepsilon, t) \\ \eta_\varepsilon(\varepsilon, t) \end{pmatrix} = \begin{pmatrix} \frac{a_2(\varepsilon)s^2 + b_2(\varepsilon)s + c_2(\varepsilon)}{s^2+bs+c} & \frac{a_3(\varepsilon)s^2 + b_3(\varepsilon)s + c_3(\varepsilon)}{s^2+bs+c} & \frac{a_4(\varepsilon)s^2 + b_4(\varepsilon)s + c_4(\varepsilon)}{s^2+bs+c} \\ \frac{a_{2\varepsilon}(\varepsilon)s^2 + b_{2\varepsilon}(\varepsilon)s + c_{2\varepsilon}(\varepsilon)}{s^2+bs+c} & \frac{a_{3\varepsilon}(\varepsilon)s^2 + b_{3\varepsilon}(\varepsilon)s + c_{3\varepsilon}(\varepsilon)}{s^2+bs+c} & \frac{a_{4\varepsilon}(\varepsilon)s^2 + b_{4\varepsilon}(\varepsilon)s + c_{4\varepsilon}(\varepsilon)}{s^2+bs+c} \end{pmatrix} \begin{pmatrix} \hat{\eta}_0(s) \\ \hat{\theta}_1(s) \\ \hat{\sigma}_1(s) \end{pmatrix} \quad \{9\}$$

Define $\eta_0 \in \mathbb{R}$ as the disturbance in the system, $\phi_d(t) \in \mathbb{R}^2$ as the state variables whose governing equation has disturbance η_0 as input, $\phi_c(t) \in \mathbb{R}^4$ as the states whose governing equations has control actions represented by $u(t) = [\theta_1(t) \;\; \sigma_1(t)]^T \in \mathbb{R}^2$ as inputs, and $Y(\varepsilon, t) = [\eta(\varepsilon, t) \;\; \eta_\varepsilon(\varepsilon, t)]^T \in \mathbb{R}^2$ as the output vector. We can express the system equations as

$$\begin{aligned} \dot{\phi}_d(t) &= A_d \phi_d(t) + B_d \eta_0(t) \\ Y_d(\varepsilon, t) &= C_d(\varepsilon)\phi_d(t) + D_d(\varepsilon)\eta_0(t) \end{aligned} \quad \{10\}$$

$$\begin{aligned}\dot{\phi}_c(t) =& A_c\phi(t) + B_c u(t)\\ Y_c(\varepsilon,t) =& C_c(\varepsilon_1)\phi_c(t) + D_c(\varepsilon)u(t)\end{aligned} \qquad \{11\}$$

$$Y(\varepsilon,t) = Y_d(\varepsilon,t) + Y_c(\varepsilon,t) \qquad \{12\}$$

where

$$A_d = \begin{pmatrix} 0 & 1 \\ -c & -b \end{pmatrix}, \quad B_d = (0\ 1)^T, \quad C_d(\varepsilon) = \begin{pmatrix} c_2(\varepsilon) - a_2(\varepsilon)c & b_2(\varepsilon) - a_2(\varepsilon)b \\ c_{2\varepsilon}(\varepsilon) - a_{2\varepsilon}(\varepsilon)c & b_{2\varepsilon}(\varepsilon) - a_{2\varepsilon}(\varepsilon)b \end{pmatrix},$$

$$A_c = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -c & -b & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -c & -b \end{pmatrix}, \quad C_c(\varepsilon) = \begin{pmatrix} c_3(\varepsilon) - a_3(\varepsilon)c & c_{3\varepsilon}(\varepsilon) - a_{3\varepsilon}(\varepsilon)c \\ -a_3(\varepsilon)b & -a_{3\varepsilon}(\varepsilon)b \\ -a_4(\varepsilon)c & -a_{4\varepsilon}(\varepsilon)c \\ b_4(\varepsilon) - a_4(\varepsilon)b & b_{4\varepsilon}(\varepsilon) - a_{4\varepsilon}(\varepsilon)b \end{pmatrix}^T, \quad B_c = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$D_d(\varepsilon) = \begin{pmatrix} a_2(\varepsilon) \\ a_{2\varepsilon}(\varepsilon) \end{pmatrix}, \quad D_c(\varepsilon) = \begin{pmatrix} a_3(\varepsilon) & a_4(\varepsilon) \\ a_{3\varepsilon}(\varepsilon) & a_{4\varepsilon}(\varepsilon) \end{pmatrix}$$

One can consider the overall system to be composed of two dynamical systems in a span. The first system, Equation {10}, provides evolution of the downstream lateral oscillation due to propagation of the disturbance. The second system, expressed by Equation {11}, represents the upstream propagation of the control actions from the guide roller. The spatially dependent output is the result of the combination of both of these effects. Note that the spatial dependency is reflected only in the output matrices $C_i(\varepsilon)$ and $D_i(\varepsilon)$. As an initial step, we will assume that one can measure the full output (lateral position and slope) at a spatial point of interest (ε_c) along the span. In what follows, we will consider a Proportional Integral (PI) controller to regulate the web position and slope at the given point, ε_c. The uncontrollable modes of the system are the two modes of the disturbance system.

The PI controller can be expressed as

$$u(t) = K_p Y(\varepsilon_c, t) + K_i \int_0^t Y(\varepsilon_c, \tau) d\tau$$

where

$$K_p = \begin{pmatrix} kp_1 & kp_2 \\ kp_3 & kp_4 \end{pmatrix}, \quad K_i = \begin{pmatrix} ki_1 & ki_2 \\ ki_1 & ki_2 \end{pmatrix}$$

Substituting this control input Eqs. {10}, {11} and {12}, the closed loop dynamics are

$$\begin{aligned}\begin{pmatrix} \dot{\phi}_c(t) \\ \dot{\phi}_d(t) \end{pmatrix} &= \begin{pmatrix} A_1 & A_2 & A_3 \\ \mathbf{0} & \mathbf{0} & A_d \end{pmatrix} \begin{pmatrix} \phi_c(t) \\ \phi_d(t) \end{pmatrix} + \begin{pmatrix} B_c(I - K_p D_c(\varepsilon_c))^{-1} K_p D_d(\varepsilon_c) \\ B_d \end{pmatrix} \eta_0(t) \\ Y(\varepsilon,t) &= \begin{pmatrix} C_1 & C_3 \end{pmatrix} \begin{pmatrix} \phi_c(t) \\ \phi_d(t) \end{pmatrix} + \left(D_d(\varepsilon) + D_c(\varepsilon)(I - K_p D_c(\varepsilon_c))^{-1} K_p D_d(\varepsilon_c)\right) \eta_0(t)\end{aligned} \qquad \{13\}$$

where the matrix I is the 2×2 identity matrix, $\mathbf{0}$ are zero matrices of appropriate dimensions, the superscript -1 denotes the inverse of a matrix, and

$$A_1 = A_c + B_c(I - K_p D_c(\varepsilon_c))^{-1} K_p C_c(\varepsilon_c), \quad A_2 = B_c(I - K_p D_c(\varepsilon_c))^{-1},$$
$$A_3 = B_c(I - K_p D_c(\varepsilon_c))^{-1} C_d(\varepsilon_c), \quad C_1 = C_c(\varepsilon) + D_c(\varepsilon_c)(I - K_p D_c(\varepsilon_c))^{-1} K_p C_c(\varepsilon),$$
$$C_2 = D_c(\varepsilon)(I - K_p D_c(\varepsilon_c))^{-1},$$
$$C_3 = C_d(\varepsilon) + D_c(\varepsilon)(I - K_p D_c(\varepsilon_c))^{-1} K_p C_d(\varepsilon_c).$$

4 SIMULATIONS WITH MIMO PI CONTROLLER

The goal is to select the gains of the matrices K_p and K_i such that the system, represented by Eq. {13}, is stable and the output for a given point in the span ε_c is regulated to zero. To visualize this, we have selected as control point ε_c=0.3, and tuned the gains until regulation at this point is obtained, the parameter data for the simulation is presented in Table 1. In all the simulations, a disturbance,

Definition	**Symbol**	**Value**	**Units**
Span Length	L	3.833 (1.1684)	$ft(m)$
Tension	T	10 (44.48)	$lbf(N)$
Transport Speed	v	500 (2.54)	$ft/min(m/s)$
Web width	w	5.4 (137.16)	in (mm)
Web thickness	h	0.005 (0.127)	in (mm)
Young's Module	E	0.40466 (2.76×10^9)	Mpsi (Pa)

Table 1 – Parameter values used in the simulations

$y_0 = 0.002\sin(3t)$, at the upstream roller is employed to evaluate the response. Figure 2 shows the open loop response of the system without control action. In this figure the evolution of the disturbance and the response of the control point (ε_c) are presented, as well as the slope at the control point.

Figure 3 shows the response of the system with pure guide roller rotation as control action to the disturbance. This rotation is generated by a PI controller, based on the feedback of a edge sensor at the control point. The web lateral response improves at the expense of the slope. However, the response is such that we cannot achieve regulation of either web lateral position or web slope to zero.

Figure 4 shows the response of the system with guide roller displacement only as control action in response to the disturbance. Similarly, this action is generated by the PI controller, based on the feedback of a edge sensor at the control point. This controller admits larger gains, and as the gain increases, so does the accuracy of the regulation of the web lateral position to zero. As in the previous case, the web lateral response improves at the expense of the slope. From these two scenarios, one can gather that a single control action may be insufficient to regulate both outputs to zero.

In Figure 5, the response of the system to the sinusoidal disturbance $y_0 = 0.002\sin(3t)$ and with two independent control actions (guide roller displacement and rotation), with PI controllers based on the measurements of an edge sensor at ε_c, is presented. As discussed before, it can be seen that the web position is regulated to zero, but the slope is not, because of only a partial measurement of the web lateral position.

In Figure 6 only the rotation of the guide roller is considered as the control action, but the feedback of two sensors is used, the first sensor measures the web lateral position, and the other sensor the web slope, both at the control point ε_c. It can be observed that an improvement in the regulation of the slope is achieved.

Similarly to the previous case, we proceed with the guide roller displacement as the only control action. The control action is the result of two PI controllers, where one controller uses the information of an edge sensor and the second controller uses the measurements of the web slope at the control point. The results are presented in Figure 7. The regulation of the web lateral position shows an improvement when compared to the regulation using the guide roller rotation as control action, but the slope is not regulated.

One may conclude that the combination of two decoupled control actions (guide roller displacement and rotation) with feedback of both web lateral position and slope, since a trade off between web lateral position and slope could be achieved. Figure 8 shows the response of the system to this type of control action, where an improvement with respect to the previous controllers is not achieved.

The best results for regulation of the web position and slope is when two independent control actions are used, the guide roller displacement action is based on the web lateral position feedback, while the guide roller angle is based on the web slope. Figure 9 shows the result of this combination.

5 ANALYSIS FOR OBSERVER DESIGN

From the previous section, it is evident that regulation of the web lateral position and web slope require independent measurements of both lateral position and slope at the control point. However, with traditional edge sensors one can only measure the lateral web position. In this Section, we will consider the problem of estimation of the web slope within the span when measurements from several edge sensors installed within the span are available. To design an observer, we will first define the following:

$$\phi(t) = [\phi_d(t)\ \ \phi_c(t)]^T, \quad A = \begin{pmatrix} A_d & 0 \\ 0 & A_c \end{pmatrix}, \quad B = [0\ \ B_c]^T; \quad E = [B_d\ \ 0], \quad C(\varepsilon) = [C_d(\varepsilon)\ \ C_c(\varepsilon)],$$

$$C_m = ([1\ \ 0]C(\varepsilon_{m1})\ \ [1\ \ 0]C(\varepsilon_{m2})...[1\ \ 0]C(\varepsilon_{mN}))^T,\ D_{cm} = ([1\ \ 0]D_c(\varepsilon_{m1})\ \ [1\ \ 0]D_c(\varepsilon_{m2})...[1\ \ 0]D_c(\varepsilon_{mN}))^T,$$

$$D_{dm} = ([1\ \ 0]D_d(\varepsilon_{m1})\ \ [1\ \ 0]D_d(\varepsilon_{m2})...[1\ \ 0]D_d(\varepsilon_{mN}))^T.$$

Note that the structure of C_m and D_m is such that only the lateral position measurements at locations $m1, m2, \ldots, mN$ are available. Thus, the system equations for observer design can be expressed as

$$\begin{aligned} \dot{\phi}(t) &= A\phi(t) + Bu(t) + E\eta_0(t) \\ Y(\varepsilon,t) &= C(\varepsilon)\phi(t) + D_c(\varepsilon)u(t) + D_d(\varepsilon)\eta_0(t) \end{aligned} \qquad \{14\}$$

In addition, one can consider the controlled output to be at a point ε_c within the span that could be different from the measurement points within the span. The equation for the controlled ouput ($Y(\varepsilon_c,t)$ and the measured output ($Z(t)$) are given by

$$Y(\varepsilon_c,t) = Y_d(\varepsilon_c,t) + Y_c(\varepsilon_c,t), \tag{15}$$
$$Z(t) = C_m\phi(t) + D_{cm}u(t) + D_{dm}\eta_0(t). \tag{16}$$

The system is observable when there are three edge sensors within the span, i.e., $N = 3$.

We consider the following Luenberger observer to obtain the estimate $\tilde{\phi}(t)$ for the state of the system:

$$\begin{aligned}\dot{\tilde{\phi}}(t) &= A\tilde{\phi}(t) + Bu(t) + E\eta_0(t) + L(Z(t) - \tilde{Z}(t))\\ Y(\varepsilon,t) &= C(\varepsilon)\tilde{\phi}(t) + D_c(\varepsilon)u(t) + D_d(\varepsilon)\eta_0(t)\\ \tilde{Y}(\varepsilon_c,t) &= \tilde{Y}_d(\varepsilon_c,t) + \tilde{Y}_c(\varepsilon_c,t)\\ \tilde{Z}(t) &= C_m\tilde{\phi}(t) + D_{cm}u(t) + D_{dm}\eta_0(t)\end{aligned} \tag{17}$$

Defining the error between the state and the estimated state as $e_\phi(t) = \phi(t) - \tilde{\phi}(t)$, where $\tilde{\phi}(t)$ is the estimated state, and the PI controller from the previous section, we obtain

$$\begin{aligned}\dot{\phi}_{ext}(t) &= \begin{pmatrix} A_{cl1} & A_{cl2} & A_{cl}3 \\ \mathbf{0} & \mathbf{0} & A - LC(\varepsilon_m)\end{pmatrix}\phi_{ext}(t) + \begin{pmatrix} B\check{K}K_pD_d(\varepsilon_c)+E \\ \mathbf{0}\end{pmatrix}\eta_0(t)\\ Y(\varepsilon,t) &= \begin{pmatrix} C_{cl1} & C_{cl3}\end{pmatrix}\phi_{ext}(t) + \left(D_d(\varepsilon) + D_c(\varepsilon)\check{K}K_pD_d(\varepsilon_c)\right)\eta_0(t)\end{aligned} \tag{18}$$

where

$$\phi_{ext(t)} = [\phi(t)\ \ e_\phi(t)]^T, \quad \check{K} = (I - K_pD_c(\varepsilon_c))^{-1}, \quad A_{cl1} = A + B\check{K}K_pC(\varepsilon_c), \quad A_{cl2} = B\check{K},$$
$$A_{cl3} = -B\check{K}C(\varepsilon_c), \quad C_{cl1} = C(\varepsilon) + D_c(\varepsilon)\check{K}K_pC(\varepsilon_c), \quad C_{cl3} = -D_c(\varepsilon)\check{K}K_pC(\varepsilon_c)$$

One can assign the poles of the observer and the controller independently and combine the controller and observer designs (separation principle).

Although we require at least three edge sensors, it is not clear where to place these sensors within the span. In the following, we propose an optimization method to determine the location of the sensors based on: (1) placing one sensor at the control point ε_c and (2) minimizing the magnitude of the estimation error. This means that we have to determine two other sensor locations. We want to minimize the magnitude of the estimation error $e_z(t) = Z(t) - \hat{Z}(t)$, which can be expressed $e_z(t) = C_m e_\phi(t)$. Since C_m is the only term that depends on the spatially position, we can perform the minimization with respect to C_m only. By taking the derivative of the Frobenius norm of C_m with respect to ε_{m1} and equating to zero, we immediately obtain the position of one sensor to be at $\varepsilon = 1$. The location of the third sensor is obtained by considering the minimum value of the norm of C_m by fixing the locations of the other two sensors at the control point (ε_c) and at the end of the span ($\varepsilon = 1$). With this approach, the resulting third location of the sensor is when ε is close to zero, i.e., at the beginning of the span. Therefore, by using this approach, we obtained the three locations for the sensors are at the start of the span, at the control point, and at the end of the span.

Figure 10 shows the evolution of the state errors $e_\phi(t)$ in time when we consider the following sensor locations: $\varepsilon_{m1} = 0.1, \varepsilon_c = 0.3, \varepsilon_{m2} = 1$. One can see that the convergence is relatively fast, with the selected gains and sensor positions.

6 CONCLUDING REMARKS

In this paper, we have presented a state space representation of the spatially dependent transfer functions for web lateral dynamics. Based on this representation, we have considered the use of independent rotation and translations control actions for the guide roller. We observed that the best regulation results for web lateral position and slope at a specific point within the span is obtained when we independently control the rotation and translation of the guide roller. Based on the spatially dependent model, we have proposed an observer to estimate the states of the system. In particular, we utilize three lateral sensor measurements within a span to estimate the web slope. Future work will consider development and evaluation of various model-based control strategies.

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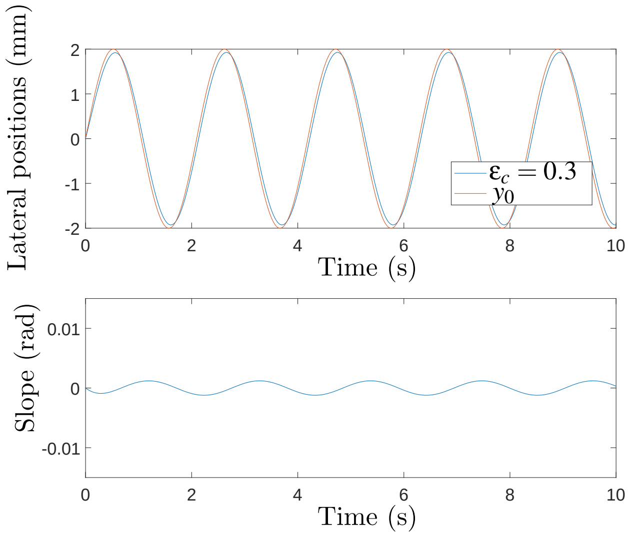

Figure 2 – Evolution of the open loop dynamics for $\varepsilon_c = 0.3$ to a disturbance of $y_0 = 0.002\sin(3t)$

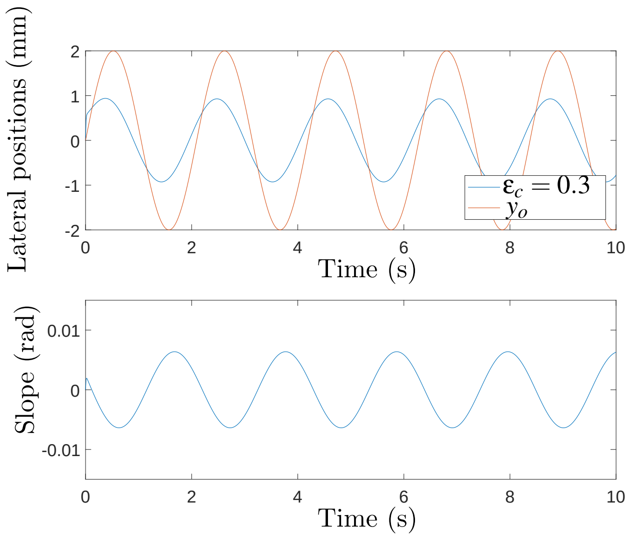

Figure 3 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with $\theta_L(t)$ as control action, with a PI control, based on edge sensor measurement, disturbance of $y_0 = 0.002\sin(3t)$

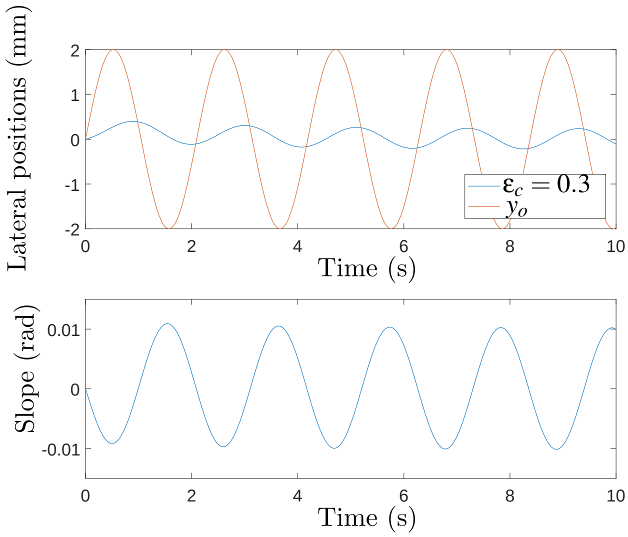

Figure 4 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with $z_L(t)$ as control action, with a PI control, based on edge sensor measurement, disturbance of $y_0 = 0.002\sin(3t)$

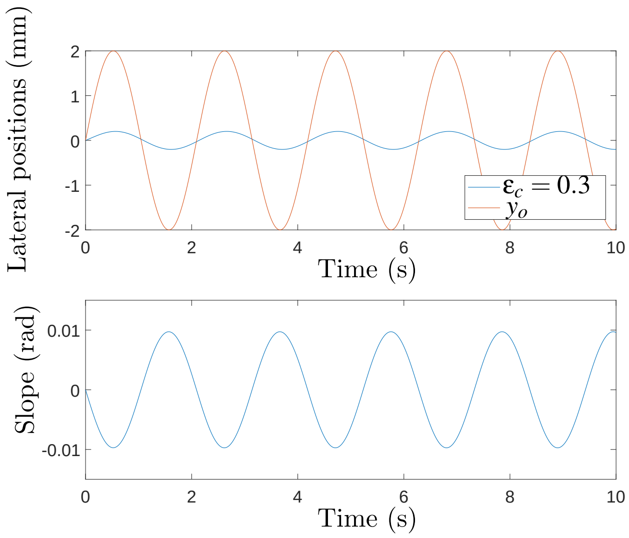

Figure 5 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with θ_L and $z_L(t)$ as control actions, with two PI controllers, based on edge sensor measurement, disturbance of $y_0 = 0.002\sin(3t)$

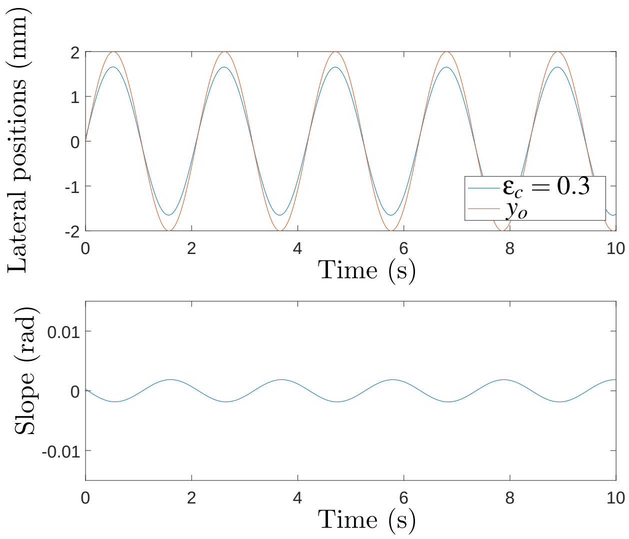

Figure 6 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with θ_L as control actions, with two PI controllers, based on edge and slope sensor measurements, disturbance of $y_0 = 0.002\sin(3t)$

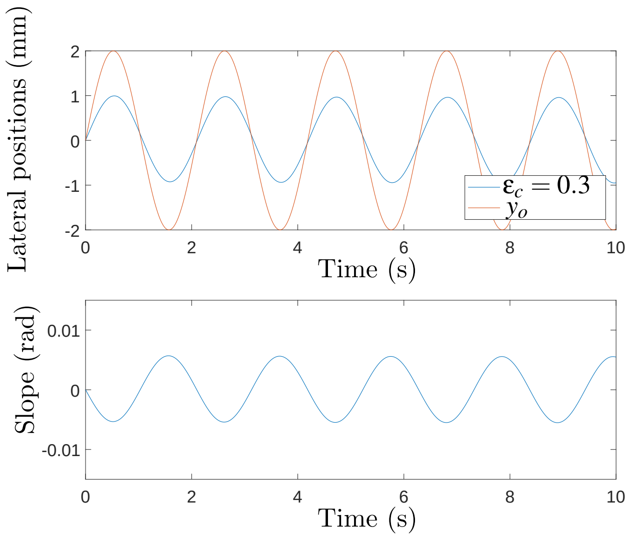

Figure 7 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with z_L as control actions, with two PI controllers, based on edge and slope sensor measurements, disturbance of $y_0 = 0.002\sin(3t)$

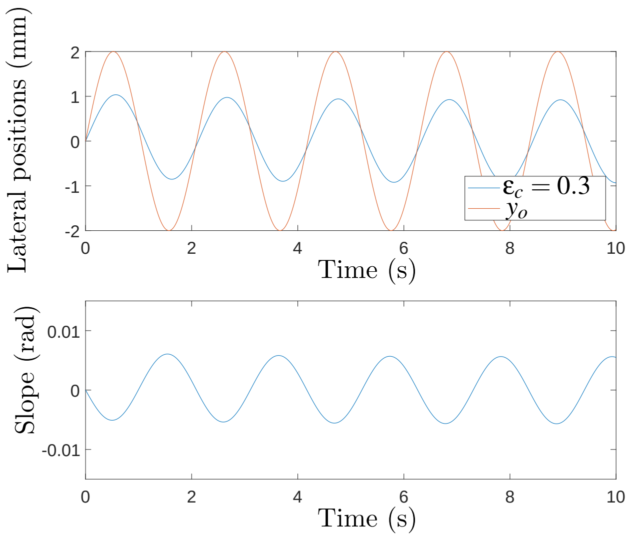

Figure 8 – Evolution of the web lateral dynamics at $\varepsilon_c = 0.3$, with θ_L and z_L as control actions, with two PI controllers, based on edge and slope sensor measurements, disturbance of $y_0 = 0.002\sin(3t)$

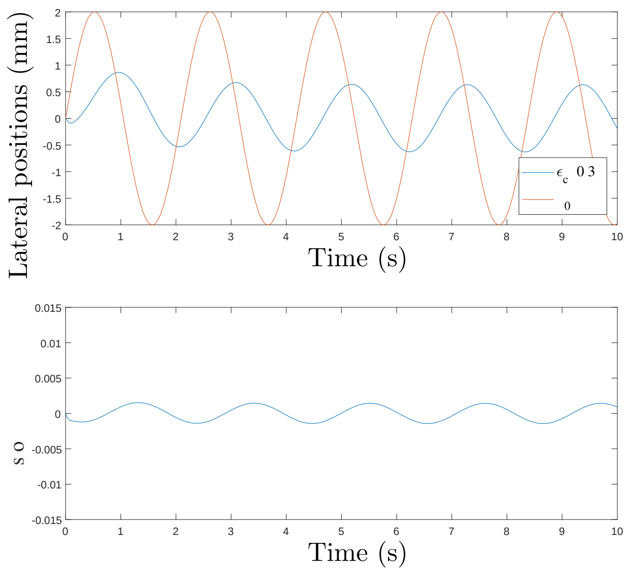

Figure 9 – Evolution of different points within the span, with θ_L and z_L as control actions, with independent PI controllers, based on edge and slope sensor measurements, disturbance of $y_0 = 0.002\sin(3t)$

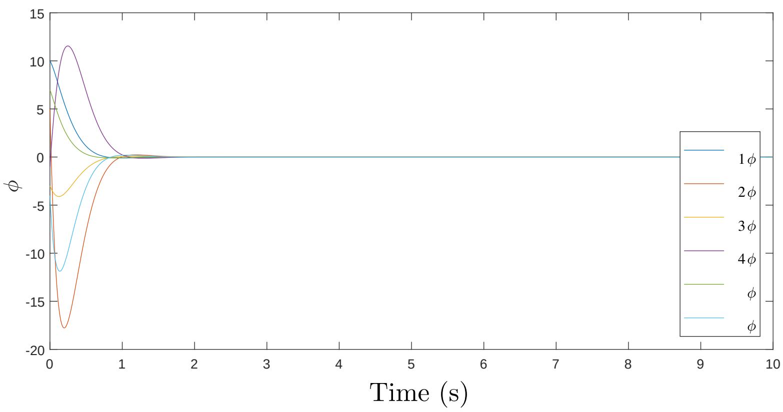

Figure 10 – Evolution of the estimated state errors